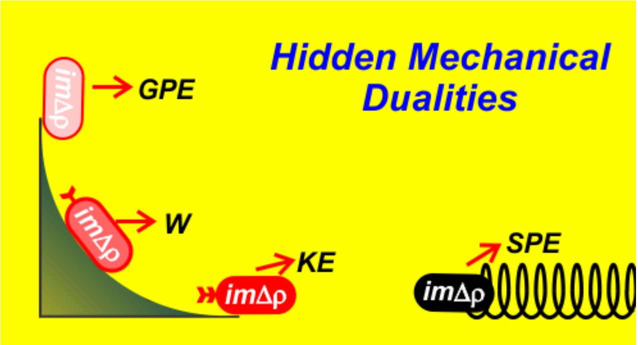
Six Duality Laws of
Momentum and Energy

Guide for improving standards.



1. First Duality Law of Momentum-Energy Coexistence

Du-Ane Du

www.Wacky1301SCI.com, *"Looking at serious science, sideways!"*

Mathematical Duality Law #1A: Momentum-impulse mathematically coexists with all forms of energy-work. For example, kinetic energy is the antiderivative of momentum, work done is the antiderivative of impulse used, Gravitational Potential Energy is the antiderivative of Gravitational Potential Impulse, and Spring Potential Energy is the antiderivative of Spring Potential Impulse. Mathematically, energy-work cannot exist without the presence of momentum-impulse, and it is often difficult to detect if a physical event is being caused by energy transfer or by momentum transfer.

The purpose of this series of advanced articles is to codify the mathematical duality between mechanical energy and momentum. In the 1850's a scientific conference voted into existence a theoretical "law of conservation of energy," and with that vote, they tried to connect numerous forms of energy together.

"Unfortunately," Devil's Advocate says, "while most scientists believe the theoretical energy connections exist, many of the connections have never actually been proven."

That's true! For example, electrical energy and calorie-energy are standard linearized forms of energy. Work done obeys the rules of multi-linear mathematics, while GPE, KE, WE, and SPE are multi-parabolic in nature. (See Article 3, Third Duality Law of non-Standard Energy Sizes and Varying Productivity.) Many of the theoretical connections between linearized energy and multi-parabolic energy have not been verified.

Because different forms of energy have different mathematical properties, this advanced article will focus primarily on mechanical energy, and any conclusions developed will pertain primarily to mechanical energy. (Articles 2 – 6 can be found at www.Wacky1301SCI.com. Wacky1301SCI also includes simplified discussions of these topics.)

This exploration of the momentum-energy duality will focus on (1) the momentum-KE duality, (2) the impulse-WE duality, (3) the Gravitational Potential Impulse-GPE duality,

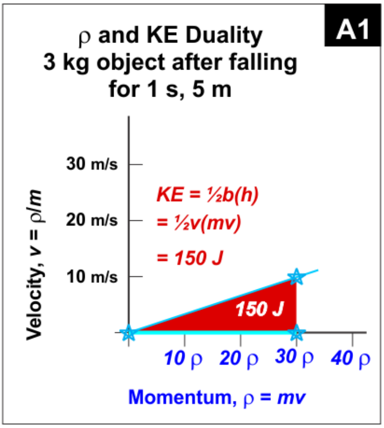
(4) the Spring Potential Impulse-SPE duality, and (5) the importance of the pendulum perspective in experimental measurements.

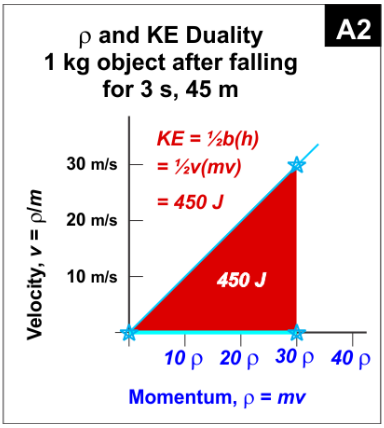
Devil's Advocate says, "Think of this article as an opportunity to expand your awareness—like reading a creative novel: Consider alternate views, empathize with other perspectives, validate new ideas, and expand your understanding of the world around you."

Momentum-KE Mathematical Duality

Symbols
$im\Delta\rho$ – impulse
10 ρ = 10 kgm/s
10 ρ = 10 N*s

Possibly the oldest known mathematical duality is the one which connects kinetic energy with momentum. Simply put, kinetic energy is the anti-derivative of momentum. This duality is easily visualized by looking at the velocity-momentum graphs of two objects moving forward with 30 kgm/s of momentum:





Looking at the first X-axis, note that the graph is following the custom of using the symbol *rho* to as a unit indicating momentums of momentum, such that:

$$10\rho = 10\ kgm/s$$

Devil's Advocate says, "This follows a practice introduced by Mr. Du in his edu-novel [*Murdered Energy Mysteries*] (Amazon, 2018)."

Precisely, using *rho* as the unit of momentum simplifies the notation.

In both graphs, the Y-axis indicates the velocity of the moving object. One cannot give an object more velocity. To increase an objects velocity, a force is applied to the side of the object, increasing the momentum, and the additional momentum causes a corresponding increase in the velocity. Velocity, therefore, is physically dependent on momentum, and it is a mathematical function of momentum. Momentum is the independent variable, and velocity is dependent.

The slope of each graph is the inverse of the mass: 1/*m*.

Finally, the area under the data line can be found as follows:

$$anti\text{-}derivitive = \frac{1}{2}bh$$

$$anti\text{-}derivitive = \frac{1}{2}(mv)v$$

$$\boldsymbol{KE = anti\text{-}derivitive = \frac{1}{2}mv^2}$$

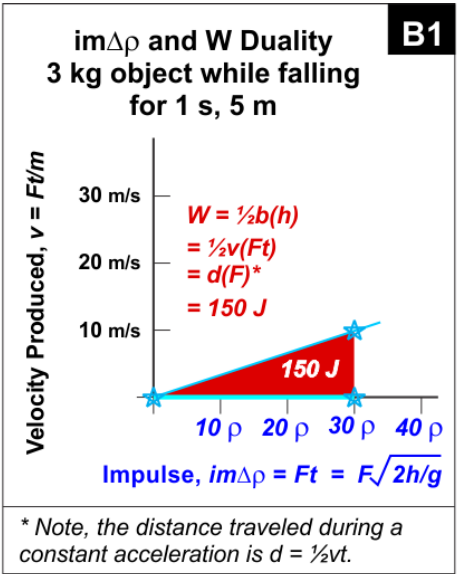
The area under the data line is the kinetic energy. This is an integral, and the anti-derivative of momentum is $\frac{1}{2}mv^2$—kinetic energy. In fact, all kinetic energy formulas are integrals of velocity-momentum graphs.

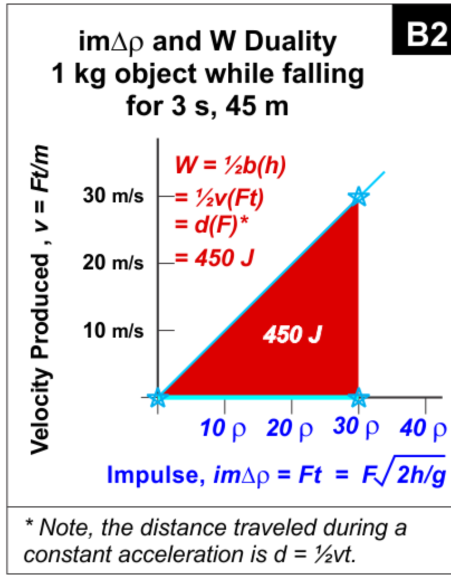
According to our graphs, kinetic energy is a joint function of an object's momentum and velocity, it is also a second-order function of the object's momentum.

"Amazing," Devil's Advocate says, "Examining these velocity-momentum graph clearly demonstrates that kinetic energy cannot exist without the presence of momentum, they are a mathematical duality."

Impulse-WE Mathematical Duality

A similar mathematical duality exists between mechanical work and impulse. To visualize this duality, we examine the same two objects as they experience a gravitational acceleration that gives them 30 kgm/s of momentum:





This time the X-axis is measured in momentums of impulse-used (ie. produced by gravity). It may be worth noting that the 10 kgm/s of impulse is the same thing as a transfer of 10 ρ of momentum (from the earth to the falling object). In terms of force, the 1 kg object is experiencing a force of 1 N for 3 s, which is an impulse of 30 ρ. Similarly, the 3 kg object is experiencing a force of 10 N for 1 s, which is also an impulse of 30 ρ.

When gravity or other acceleration is involved, the impulse equation can be changed to a height- or distance-based equation, by substituting in the distance equation, as follows:

$$d = \frac{1}{2}at^2$$
$$t^2 = 2d/a$$
$$t = \sqrt{2d/a}$$

$$t = \sqrt{2h/g}$$
$$im\Delta\rho = Ft$$
$$im\Delta\rho = F\sqrt{2h/g}$$

When the impulse-used is divided by the mass of the object, the calculation produces a value for the resultant velocity.

"This means," Devil's Advocate says, "the resultant velocity is a function of the impulse used."

Correct, what about the integral. To derive this integral, first note that the impulse (X) is causing the object to accelerate from zero to a new velocity (Y). The equation for distance traveled during an acceleration is:

$$d = \frac{v_2 - 0}{2}t$$
$$d = \frac{1}{2}vt$$

The equation for the area under the data line becomes:

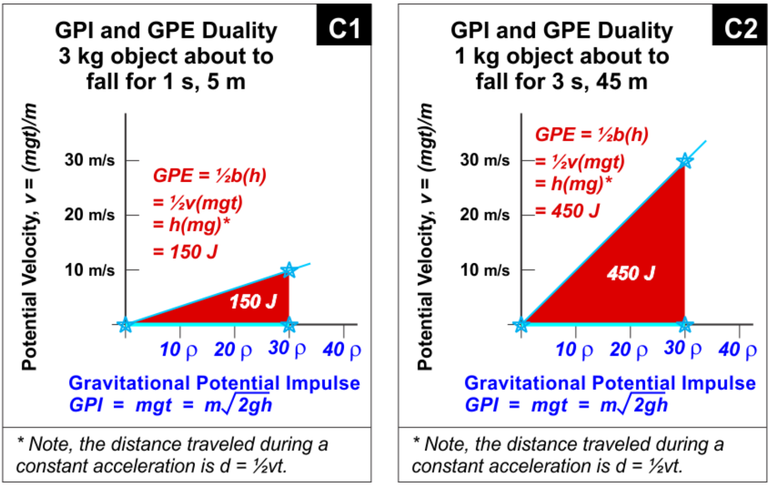
$$anti\text{-}derivative = 1/2bh$$
$$anti\text{-}derivative = 1/2v(Ft)$$
$$anti\text{-}derivative = (1/2vt)F$$
$$d = \frac{1}{2}vt$$
$$Work\text{-}energy = anti\text{-}derivative = dF$$

"This means," Devil's Advocate says, "work-energy is the anti-derivative of impulse used."

Precisely, work is a joint function of impulse and velocity, and work is a second-order function of impulse-used. This is an undeniable duality such that work-done can never exist without impulse-used.

Gravitational Potential Impulse-GPE Duality

The third duality to be examined is the GPI-GPE duality. This duality is easily visualized by examining the velocity-potential impulse graphs for the two objects under consideration:



As before, the distance-based equation for Gravitational Potential Impulse is derived using the distance-time equation:

$$d = \frac{1}{2}at^2$$

$$t = \sqrt{2h/g}$$
$$GPI = mgh$$
$$GPI = mg\sqrt{2h/g}$$
$$\boldsymbol{GPI = m\sqrt{2gh}}$$

The two equations for potential velocity can be developed simply by dividing the GPI equations by the mass being examined:

$$Potential\ velocity,\ v = \frac{mgt}{m} = gt$$
$$Potential\ velocity,\ v = \frac{m\sqrt{2gh}}{m} = \sqrt{2gh}$$

Finally, the area under the data line can be found using:

$$anti\text{-}derivative = \frac{1}{2}bh$$
$$anti\text{-}derivative = \frac{1}{2}(mgt)(v)$$
$$anti\text{-}derivative = (\frac{1}{2}vt)(mg)$$
$$h = \frac{1}{2}vt$$
$$\boldsymbol{GPE = anti\text{-}derivative = h(mg) = mgh}$$

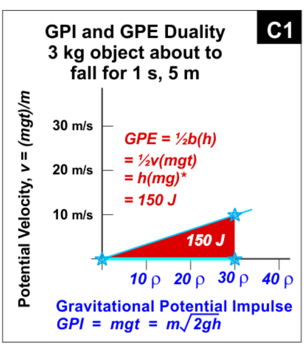
"As before," Devil's Advocate says, "Gravitational Potential Energy is the anti-derivative of Gravitational Potential Impulse."

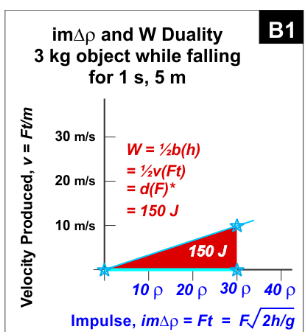
Exactly, GPE is a joint function of potential impulse and potential velocity, and GPE is a second-order function of potential impulse. As a result, Potential Energy and Potential Impulse are a mathematical duality, they are often confused during experiments, and it is often difficult to tell if an experimental result is being caused by one or the other.

Momentum-Impulse Equivalencies in Action

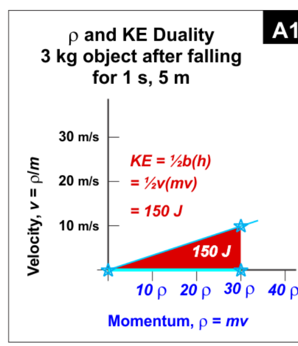
If the objects under consideration are being held above a spring, they will fall and hit the spring. According to momentum theory, the objects have Gravitational Potential Impulse before they fall. As they fall, they experience an impulse, and at the end of the fall they have a final momentum that is equal to the predicted Potential Impulse.

Graphs of the 3 kg object before, during and after the fall are as follows:





* Note, the distance traveled during a constant acceleration is d = ½vt.



This demonstrates the momentum equivalencies:

$$GPI \cong im\Delta\rho \ during \ \cong momentum \ after$$
$$mgt = Ft = \Delta mv$$
$$30\ \rho = 30\ \rho = 30\ \rho$$

The area under the three graphs indicates the energy equivalencies:

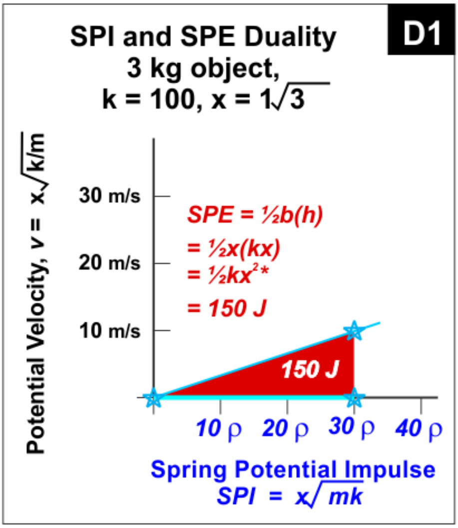
$$GPE \cong Work\ during \cong KE\ after$$
$$mgh = Fd = \frac{1}{2}mv^2$$
$$150\,J = 150\,J = 150\,J$$

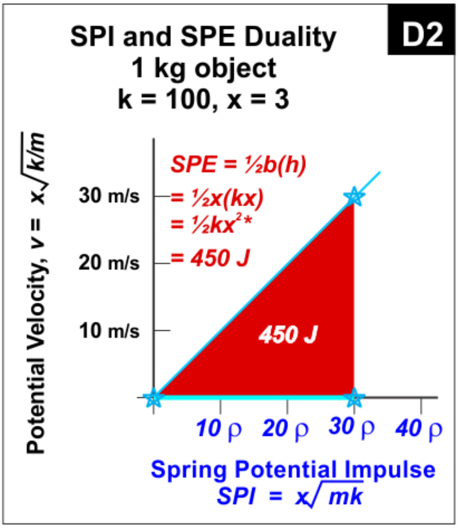
"These momentum-energy equivalencies are mathematical dualities," Devil's Advocate says. "As a result, it is often very difficult to identify which equality is occurring in which experiment." (See situational creation and situational destruction, Article 3 and Article 4.)

Spring Potential Impulse-SPE Duality

Continuing with this example, when the falling objects reach their indicated momentums, they contact a spring—and they transmit their momentum into the spring, causing it to compress, and slowing the objects to a stop (zero visible momentum).

When compressed, the springs and the objects together have a Spring Potential Impulse. The SPI for the two objects can be seen in the following graphs:





The equation for Spring Potential Impulse is most easily derived from the traditional energy equations:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v^2 = \frac{k}{m}x^2$$

$$\boldsymbol{potential\ v = x\sqrt{\frac{k}{m}}}$$

$$\boldsymbol{m}v = \boldsymbol{m}x\sqrt{\frac{k}{m}}$$

$$\boldsymbol{SPI = x\sqrt{mk}}$$

“Interesting,” Devil’s Advocate says, “This equation for Spring Potential Impulse contains an efficiency component ($\sqrt{m}$).” (This efficiency component and other spring-impulse equations are examined in *Murdered Energy Mysteries*, Chapters E06 and E07.)

Excellent point. Note that the equation for potential velocity was the third step in the above derivation. The equation for the area below the line becomes:

$$anti\text{-}derivative = \frac{1}{2}bh$$

$$anti\text{-}derivative = \frac{1}{2}(x\sqrt{\frac{k}{m}})(x\sqrt{mk})$$

$$anti\text{-}derivative = \frac{1}{2}xx\sqrt{\frac{k}{m}}\sqrt{mk}$$

$$\boldsymbol{SPE = anti\text{-}derivative = \frac{1}{2}kx^2}$$

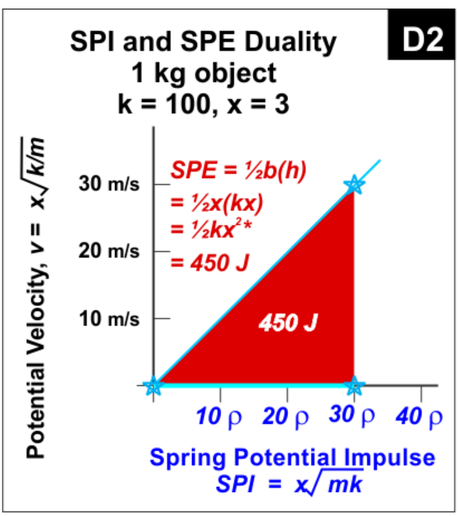
"As you would now expect," Devil's Advocate says, "SPI and SPE clearly are a mathematical duality."

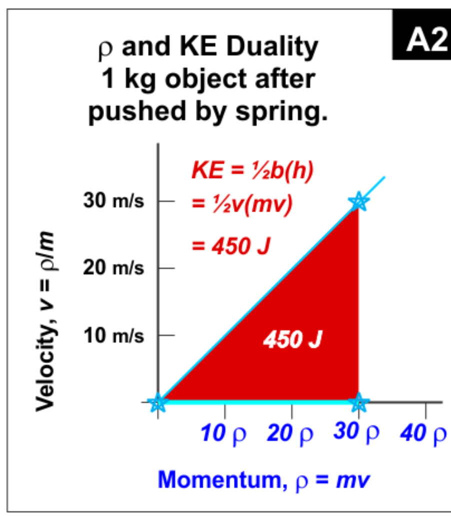
Very true, Spring Potential Energy is the anti-derivative of Spring Potential Impulse. SPE is a joint function of potential impulse and potential velocity, and SPE is a second-order function of potential impulse.

"As a result, " Devil's Advocate says, "SPE and SPI are easily confused, and it is often difficult to tell if an experimental result is being caused by one or the other."

In fact, most scientists even go so far as to deny that Spring Potential Impulse exists. (Many also deny the existence of Gravitational Potential Impulse and other forms of momentum/impulse.)

Continuing with our analysis: When the springs release, the release of Spring Potential Impulse will cause the objects to gain momentum in the opposite direction. The relationship between the Spring Potential Impulse before the release, and the amount of momentum received by the 1 kg object be seen in the following graphs of the 1 kg object:





The SPI-becomes-momentum equivalency, expands the cycle of momentum equivalencies to:

$$
\begin{array}{lllll}
GPI & \cong im\Delta\rho\ during & \cong \rho\ after & \cong SPI \\
mgt & = \ Ft & = \Delta mv & = x\sqrt{mk} \\
m\sqrt{2gh} & = F\sqrt{2h/g} & = \Delta mv & = d\sqrt{mk} \\
30\ \rho & = \ 30\ \rho & = 30\ \rho & = 30\ \rho
\end{array}
$$

"However," Devil's Advocate says, "it's important to understand that these equivalencies are only valid when the measurements are made from a stationary position, at the bottom of the fall, and at the neutral contact point between the object and the spring."

Absolutely critical! but why is the stationary perspective important?

Importance of the pendulum perspective

The momentum-impulse equivalencies and the energy-work equivalencies are only valid when measurements are made from the "pendulum perspective".

In pendulum experiments, all measurements are made with respect to a stationary point at the bottom of the pendulum swing. This position means the height, gravity, and velocity are absolute measurements, and they are linked together in the controlling equalities:

$v^2 = 2gh$ (energy-work control)

$v = \sqrt{2gh}$ (momentum-impulse control)

"According to these equations," Devil's Advocate says, "at the lowest point of its swing, the velocity of a pendulum is a joint function of the gravity and swing-height."

Precisely, and if any of the three measurements is distorted, then the controlling equalities become invalid, the momentum-impulse equivalencies become invalid, and the energy-work equivalencies become invalid.

For example, if the height is measured from a position other than the bottom of the swing, then the GPI and GPE equivalencies become invalid, and:

$$\sqrt{2g(\boldsymbol{h} \pm \boldsymbol{h_{Observer}})} \neq v$$
$$GPI = \boldsymbol{m}\sqrt{\boldsymbol{2g}(\boldsymbol{h} \pm \boldsymbol{h_O})} \neq im\Delta\rho \neq \rho$$

$$2g(\boldsymbol{h} \pm \boldsymbol{h_{Observer}}) \neq v^2$$
$$GPE = \boldsymbol{mg}(\boldsymbol{h} \pm \boldsymbol{h_O}) \neq W \neq KE$$

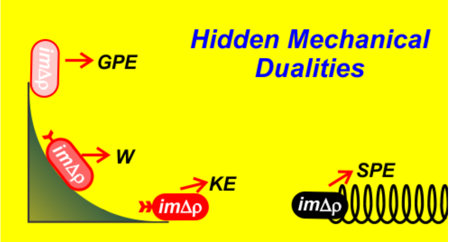
Likewise, if the velocity is measured from a moving position, then the momentum and kinetic energy equivalencies become invalid, and:

$$(\boldsymbol{v} \pm \boldsymbol{v_{Obsover}}) \neq \sqrt{2gh}$$
$$\rho = \boldsymbol{m}(\boldsymbol{v} \pm \boldsymbol{v_O}) \neq im\Delta\rho \neq GPI$$

$$(\boldsymbol{v} \pm \boldsymbol{v_{Obsover}})^{\boldsymbol{2}} \neq 2gh$$
$$KE = \boldsymbol{\frac{1}{2}m}(\boldsymbol{v} \pm \boldsymbol{v_O})^{\boldsymbol{2}} \neq W \neq GPE$$

“Clearly,” Devil’s Advocate says, “best practice requires that when a non-pendulum observation point is used, the data must be adjusted to correspond to a stationary observation point at the bottom of the swing, thus the distortion must be subtracted out of the measurement.”

(While some of this may seem obvious, educators and scientific philosophers commonly use moving reference points to measure velocity, which invalidates the concept of energy-work equivalency, etc.)



CONCLUSION 1: **First Duality Law of Momentum-Energy Coexistence:** Momentum-impulse mathematically co-exists with all forms of energy-work. For example, kinetic energy is the antiderivative of momentum, work done is the antiderivative of impulse used, Gravitational Potential Energy is the antiderivative of Gravitational Potential Impulse, and Spring Potential Energy is the anti-derivative of Spring Potential Impulse. Mathematically, energy-work cannot exist without the presence of momentum-impulse, and it is often difficult to detect if a physical event is being caused by energy transfer or by momentum transfer.

CONCLUSION 2: **Perspective Corollary #1B:** Momentum-impulse equivalency and energy-work equivalency only occur when measurements are made from the "pendulum perspective". As a result, best practice requires that all experiments be constructed and measured as if pendulums are present. Velocity must be measured with respect to a stationary reference point, and height should be measured from the bottom of the experimental event. The use of a moving perspective invalidates both the law of conservation for energy-work and the law of conservation for momentum-impulse.

CONCLUSION 3: More research needs to be done into the relationship between mechanical energy and other theoretical forms of energy. Many common beliefs may actually be philosophical myths.

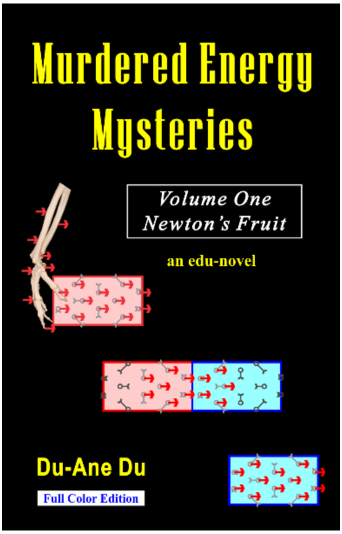
Murdered Energy Mysteries is an edu-novel that seeks to increase understanding of the various forms of momentum and momentum transfer, as well as the various forms of energy and energy transfer. The lack of understanding on the part of the scientific community is substantial, and more research needs to be done.

—Du-Ane Du, author of the edu-novel *Murdered Energy Mysteries* (Amazon, Kindle, e-book 2018, paperback 2021.)

For more information, see:

Murdered Energy Mysteries,

as well as:



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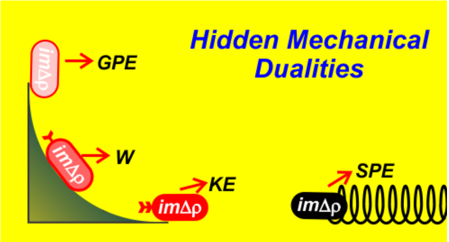
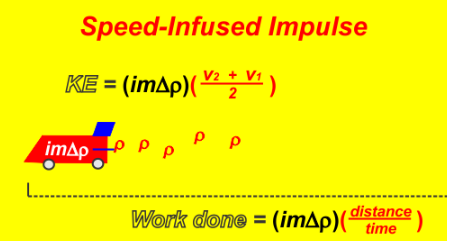
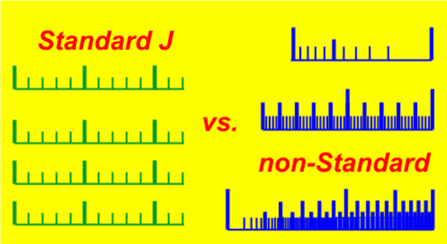
Article 1: First Duality Law of Momentum-Energy Coexistence



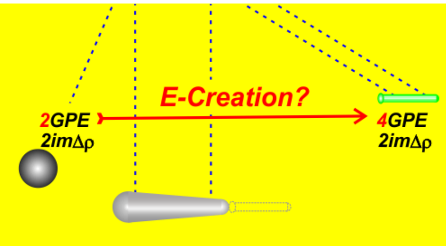
Article 2: Second Duality Law of Perceived Speedy-Impulse



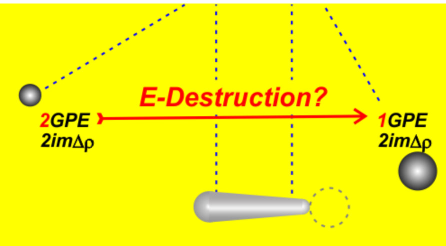
Article 3: Third Duality Law of non-Standard Energy Sizes and Varying Productivity



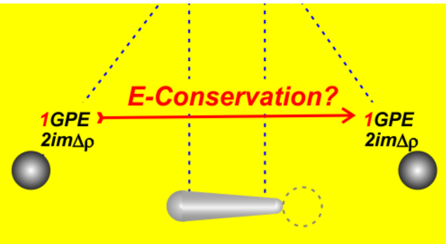
Article 4: Fourth Duality Law of Situational Energy Creation



Article 5: Fifth Duality Law of Situational Energy Destruction



Article 6: Sixth Duality Law of Situational Energy Conservation & Einstein



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